Notation: how to read a set

A set is a collection of objects, written inside braces. Small sets can be listed: $\{1, 2, 3\}$. Most sets are too big to list, so we *describe* them instead:

$$\left\{ \underbrace{x \in \mathbb{R}}_{\text{what kind of thing, and a name for it}} \;\middle|\; \underbrace{x > 0}_{\text{the test it must pass}} \right\}$$

Read the bar as "such that", then read the whole thing out loud: *"the set of all x in $\mathbb{R}$ such that x is greater than zero."* Everything left of the bar says what sort of object we are collecting and gives it a temporary name; everything right of the bar is a condition. An object is in the set exactly when it passes that test — and nothing else is.

A second form puts a recipe on the left instead of a name: $\{\, 2n \mid n \in \mathbb{Z} \,\}$ is the set of even integers. Build $2n$, and let n run over all integers.

Later this term: $\operatorname{Nul} A = \{\mathrm{x} \mid A\mathrm{x} = 0\}$ is a *test* (which vectors pass?), while $\operatorname{Col} A = \{A\mathrm{x} \mid \mathrm{x} \in \mathbb{R}^n\}$ is a *recipe* (what can we build?).

Notation you will see all semester

Symbol	Read it as	Example
$\in$	is in, belongs to	$3 \in \mathbb{Z}$
$\notin$	is not in	$\frac{1}{2} \notin \mathbb{Z}$
$\{\ \}$	the set of	$\{1, 2, 3\}$
$\mid$ (or :)	such that	$\{x \in \mathbb{R} \mid x > 0\}$
$\forall$	for all, for every	$\forall x \in \mathbb{R},\ x^2 \geq 0$
$\exists$	there exists, there is	$\exists\,\mathrm{x}$ with $A\mathrm{x} = \mathrm{b}$
$\exists!$	there is exactly one	unique solution
$\subseteq$	is a subset of	$\mathbb{Z} \subseteq \mathbb{R}$
$\varnothing$	the empty set	no elements at all

Symbol	Read it as	Example
$\mathbb{N}$	natural numbers	$1, 2, 3, \ldots$
$\mathbb{Z}$	integers	$\ldots, -1, 0, 1, \ldots$
$\mathbb{Q}$	rational numbers	$\frac{p}{q},\ q \neq 0$
$\mathbb{R}$	real numbers	$\pi,\ -\sqrt{2},\ 0.5$
$\mathbb{C}$	complex numbers	$2 - 3i$
$\mathbb{R}^n$	lists of n real numbers	$(1, 0, -4) \in \mathbb{R}^3$
$\cup,\ \cap$	union, intersection	$A \cup B,\ A \cap B$
$\Rightarrow$	implies	$x > 2 \Rightarrow x > 0$
$\iff$	if and only if	$x > 0 \iff -x < 0$

The textbook writes a colon where we write a bar — $\{\mathrm{x} : A\mathrm{x} = 0\}$ — same meaning.